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Delay Distortion in Generalized Lens-Like Media

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Explicit expressions are derived for the phase constant, the specific-group-delay constant, and the rms width of the impulse response for two-dimensional or square media having a transverse variation of index of refraction according to $n = n_1(1 - \frac{1}{2}a_u x^u - \frac{1}{2}a_r x^r)$, in which x is the transverse dimension, a_u and a_r are constants with $|a_r| \ll a_u$, and $(n - n_1) \ll 1$. Use is made of an approximation which the author has previously shown yields significant results.

The results are applied to fibers with graded-index variation, clad by an additional medium of index $n = n_1(1 - \Delta)$. The ideal index gradient, a near-parabolic profile, gives delay distortion orders of magnitude less than for the conventional fiber with a step-change in index at the core-cladding boundary. However, it is shown that several forms of 5-percent error in the ideal gradient yield improvement of the order of 50 compared with the conventional clad fiber. The delay distortion is shown to be very sensitive to the exact index distribution in the vicinity of the ideal distribution but increasingly insensitive to perturbations in the index distribution as that distribution departs more and more from the ideal.

I. INTRODUCTION

Optical fibers have assumed considerable importance for potential use as transmission media wherever wire pairs or coaxials are now used.

We give here some theory related to increasing the information capacity of such fibers.

Conventional fibers have a core of uniform index of refraction n, surrounded by a cladding of slightly lower index of refraction. The cladding serves to isolate the outer fiber surface from the optical field, which is confined to the core, thus permitting the fiber to be handled and bundled into cables without affecting the information transmission. More recently, fibers having continuously graded index of refraction, with maximum on the fiber's axis and lower values at increasing radial distances, were proposed and realized in practice.[1–3] Graded-index fibers have image-focusing properties[3] and provide modulated-carrier transmission with less delay distortion than conventional fibers having a step-index change at the core-cladding boundary.[4–6] Recent experimental work verified the potential of low delay distortion.[7] Current studies of graded-index fibers from Corning Glass Works show that total transmission losses under 10dB/kilometer and low delay distortion (approximately one nanosecond per kilometer) can be realized simultaneously in graded-index fibers.[8,9] This brings into focus the need for detailed knowledge about delay distortion related to the shape of the index gradient in the transverse plane. The following sections relate to that need. This study was done in parallel with that of D. Gloge and E. A. J. Marcatili.[10] The present paper presents an approximate analysis that may be extended to a wide variety of index distributions with closed-form solutions for the important wave-propagation constants.

Section II summarizes the approximate method of calculation, yielding in turn the phase constant, the specific-delay constant, and the impulse response.

Section III gives the solutions for the index distribution,

$$n = n_1(1 - \tfrac{1}{2}a_u x^u - \tfrac{1}{2}a_r x^r), \tag{1}$$

in which x is the radial coordinate, and a_u and a_r are arbitrary constants with $a_u > 0$, $a_u \gg |a_r|$, and $0.5\ a_u x^u \ll 1$. These conditions describe fibers of current interest.

Sections IV through VI discuss cases of interest, taken as simplifications of (1), which describe (*i*) an "ideal" index distribution, (*ii*) variations around the ideal, and (*iii*) other distributions which may result from convenient manufacturing processes.

This paper is concerned with the delay difference between signals launched simultaneously in the various propagating modes; the fiber-

output impulse response is derived assuming

(*i*) all propagating modes are excited equally at the fiber input,
(*ii*) the fiber structure is uniform along its length, yielding no mode conversion,
(*iii*) there are negligible losses or, equivalently, the same loss for all modes, and
(*iv*) material dispersion due to variation of the bulk index of refraction versus wavelength is ignored.

A very few modes very near cutoff are not accounted for; this is believed to yield negligihle error, and the same assumption was made by Gloge and Marcatili.[10]

It is found that the delay distortion of graded-index media is a very critical function of the index distrihution in the vicinity of the near-parabolic distribution which gives the lowest delay distortion. For index distributions increasingly far from the ideal, the performance is less and less sensitive to changes.

For fixed percentage error in fabrication of the index distribution, the delay distortion is *linearly* dependent on Δ, the fractional index difference between core center and the edge of the guiding region where the index becomes

$$n = n_1(1 - \Delta). \tag{2}$$

This is true even near the ideal index distribution, where previous work assuming no fabrication error indicated the delay distortion varied as Δ^2.[11]

II. OUTLINE OF THEORY

The approach taken here is based on Ref. 5, which gives an approximate method for deriving the phase constant and other relevant quantities for wave propagation in the generalized media.

We write the index n as a function of the transverse coordinate x,

$$n = n_1[1 + f(x)] \tag{3}$$

with $|f(x)| \ll 1$. For guiding media, $f(x)$ is predominantly negative; $x = 0$ at the central axis. Here we assume $f(x)$ is independent of the longitudinal coordinate. In accordance with Ref. 5 we derive a parameter a_e, which measures the transverse field width, using

$$f(a_e) = -0.1515 \left(\frac{m + 2.5}{2.5} \right)^2 \left(\frac{\lambda_0}{a_e n_1} \right)^2. \tag{4}$$

Thus the phase constant β is given hy, from Ref. 5,

$$\beta = \frac{2\pi n_1}{\lambda_0}\left\{1 - \frac{1}{32}\left(\frac{\lambda_0}{n_1 a_e}\right)^2 (m+1)^2\right\}. \tag{5}$$

The normal mode field may he considered composed of plane-wave components traveling at an angle α to the longitudinal axis; from Ref. 5,

$$\alpha = \frac{\lambda_0}{n_1}\frac{(m+1)}{4a_e} \tag{6}$$

in which m is the mode numher. The mth-order mode has $(m+1)$ extrema in the transverse cross section. The maximum angle that such plane-wave components can have is set by the fractional index difference Δ [defined in (2)]; any components with α larger than

$$\alpha_{\max} = \sqrt{2\Delta} \tag{7}$$

exceed the critical angle for total internal reflection and are unguided. Thus eqs. (7) and (6) taken together estahlish a maximum modal index $m_{\max}$, with $(m+1)_{\max}$ transverse extreme, controlled hy a_e and Δ.

The specific group delay is

$$\tau = \frac{d\beta}{d\omega}, \tag{8}$$

where $\omega = 2\pi f$ is the angular frequency. The range of values τ can assume run from the value with $m = 0$ to $\tau_{\max}$ which is

$$\tau_{\max} = \tau\big|_{(m+1)_{\max}}. \tag{9}$$

It is convenient to compare the specific group delay for the guided mode τ to that for an infinite medium of index n_1 using

$$t = \left(\tau - \frac{n_1}{c}\right). \tag{10}$$

The work of Ref. 5 related specifically to two-dimensional waveguides. We extend this here to three-dimensional guides in Cartesian coordinates. We note the relation between the propagation constants has the form

$$\epsilon_r\beta_0^2 = \beta_z^2 + \beta_{x1}^2 + \beta_{x2}^2, \tag{11}$$

where ϵ_r is the dielectric constant, β_0 is the free-space phase constant, and β_z, β_{x1}, and β_{x2} are the longitudinal and transverse wave numbers respectively. In our case, β_{x1} and β_{x2} are small compared to β_z. The value of β_z depends only on the sum of the squares of the two transverse

wave numhers $\beta_{x1}^2 + \beta_{x2}^2$; further, for a square guide of width $2a_e$ we can write

$$\beta_{x1} 2a_e = (m_a + 1)\pi, \tag{12}$$

$$\beta_{x2} 2a_e = (m_b + 1)\pi. \tag{13}$$

Thus the total modal designation is

$$\rho = (m + 1) = \sqrt{(m_a + 1)^2 + (m_b + 1)^2} \tag{14}$$

for the square guide. This replaces $(m + 1)$ in the two-dimensional guide. The maximum $(m + 1)_{\max}$ can he reached with a variety of field distrihutions given hy various values of m_a and m_b in (14).

To compute the impulse response we first note in solutions for specific group delay τ from (8) that all comhinations of modes having the same m or ρ in (14) have the *same* specific group delay. Hence the fiher output at a given delay associated with ρ will he $P(\rho)$, for equal power into the fiher in each mode, where

$$P(\rho) = \frac{\pi\rho}{2}\, d\rho \tag{15}$$

and ρ is given hy (14). The fiher impulse response as a function of time t is

$$\text{Output} = P(t) = \frac{P(\rho)}{dt} = \frac{\pi\rho}{2}\left|\frac{dt}{d\rho}\right|^{-1}. \tag{16}$$

The relation hetween t and ρ is ohtained from (10) and (8). The range of t over which (16) is valid is found hy inserting the minimum and maximum values that ρ may assume,

$$\rho_{\min} = \sqrt{2}, \tag{17}$$

$$\rho_{\max} = (m + 1)_{\max}. \tag{18}$$

For many purposes the value of $t_{\min}$ corresponding to $\rho_{\min}$ may he taken as zero since $\rho_{\max} \gg \rho_{\min}$; thus

$$t_{\min} \simeq 0, \tag{19}$$

$$t_{\max} = \left(\tau_{\max} - \frac{n_1}{c}\right). \tag{20}$$

The effect on transmission system performance is approximately the same for various shapes of the impulse response if the rms width is the same;[12] this applies in the region where the fiher impulse response degrades the system signal-power requirements hy only 1 or 2 dB. We

find the desired second moment using

$$A = \int_0^{t_{\max}} P(t)dt, \tag{21}$$

$$T = \frac{1}{A}\int_0^{t_{\max}} tP(t)dt, \tag{22}$$

$$\sigma^2 = \frac{1}{A}\int_0^{t_{\max}} t^2P(t)dt - T^2. \tag{23}$$

III. A GENERAL SOLUTION

We now give the results for the index distribution according to eq. (1). The guide has width $2a$. We specify that at $x = a$, $f(x) = -\Delta$, leading to eq. (2) and

$$a_u = \frac{2(1-\delta)\Delta}{a^u}, \tag{24}$$

$$a_r = \frac{2\delta\Delta}{a^r}, \tag{25}$$

in which δ may be either positive or negative but $|\delta| \ll 1$. Using eq. (4), we find a_e,

$$\frac{1}{a_e^2} = \left\{\frac{n_1^2 a_u}{0.303b^4\lambda_0^2}\right\}^{2/(u+2)} + \frac{2a_r n_1^2}{(u+2)0.303b^4\lambda_0^2\left\{\dfrac{n_1^2a_u}{0.303b^4\lambda_0^2}\right\}^{r/(u+2)}}. \tag{26}$$

This leads immediately to β,

$$\begin{aligned}\beta = \frac{2\pi n_1}{\lambda_0} &- \frac{\pi(m+1)^2}{16(0.1515)^{2/(u+2)}\left(\dfrac{m+2.5}{2.5}\right)^{4/(u+2)}} \\ &\times \frac{[(1-\delta)\Delta]^{2/(u+2)}}{a^{2u/(u+2)}}\left(\frac{\lambda_0}{n_1}\right)^{(u-2)/(u+2)} \\ &- 2.592 \times \frac{(0.1515)^{r/(u+2)}(m+1)^2\Delta^{(u+2-r)/(u+2)}\delta}{(u+2)\left(\dfrac{m+2.5}{2.5}\right)^{(2u+4-2r)/(u+2)}(1-\delta)^{r/(u+2)}a^{2r/(u+2)}} \\ &\times\left(\frac{\lambda_0}{n_1}\right)^{(2r-u-2)/(u+2)}, \end{aligned} \tag{27}$$

and to the number of propagating modes N,

$$N = \frac{4}{\pi}(0.7757)^{2/u}(n_1ka)^2\Delta, \tag{28}$$

in which $k = 2\pi/\lambda_0$. The maximum modal index has two useful forms,

derived from (6) and (7),

$$\frac{(m+1)^2_{\max}}{(m+2.5)^{4/(u+2)}_{\max}} = 32(0.02424)^{2/(u+2)} \left(\frac{n_1 a}{\lambda_0}\right)^{2u/(u+2)} \Delta^{u/(u+2)}, \quad (29)$$

$$m_{\max} \simeq 5.657(0.7757)^{1/u} \left(\frac{n_1 a}{\lambda_0}\right) \Delta^{\frac{1}{2}}. \quad (30)$$

The specific group delay from (8) is

$$\tau = \frac{n_1}{c}\left\{1 + \frac{(u-2)}{(u+2)} \Delta(1-\delta)^{2/(u+2)} \frac{(m+1)^2}{(m+1)^2_{\max}} \frac{(m+2.5)^{4/(u+2)}_{\max}}{(m+2.5)^{4/(u+2)}} + Q\Delta \frac{\delta}{(1-\delta)^{r/(u+2)}} \frac{(m+1)^2}{(m+1)^{2r/u}_{\max}} \frac{(m+2.5)^{4r/n(n+2)}_{\max}}{(m+2.5)^{(2u+4-2r)/(u+2)}}\right\}, \quad (31)$$

where Q is

$$Q = 2.578(0.7757)^{r/u} \frac{(2r-u-2)}{(u+2)^2}. \quad (32)$$

We can simplify τ for $m \gg 1$ to

$$\tau = \frac{n_1}{c}\left\{1 + \frac{(u-2)}{(u+2)} \Delta(1-\delta)^{2/(u+2)} \left(\frac{m}{m_{\max}}\right)^{2u/(u+2)} + Q\Delta \frac{\delta}{(1-\delta)^{r/(u+2)}} \left(\frac{m}{m_{\max}}\right)^{2r/(u+2)}\right\}. \quad (33)$$

Using (21), (22), and (23) we find the impulse response is

$$P(t)\,|_{u\neq 2} = t^{2/u}\left\{\frac{\pi(u+2)^{(2u+2)/u} m^2_{\max}}{4u(u-2)^{(u+2)/u}\Delta^{(u+2)/u}(1-\delta)^{2/u}}\right\} - t^{(r+2-u)/u}\left\{\frac{\pi(u+2)^{(2+r+2u)/u} Q\delta m^2_{\max}}{4n^2(n-2)^{(2+r+u)/u}(1-\delta)^{(r+2)/n}\Delta^{(r+2)/n}}\right\} \quad (34)$$

and the rms width of the impulse response is

$$\sigma = \frac{n_1}{c}\Delta(u-2)\left(\frac{1}{3u+2} - \frac{(u+2)}{(2u+2)^2}\right)^{\frac{1}{2}} \quad (35)$$

for the case where $a_r = 0$. The minimum allowed value of t in (34) is

$$t_{\min} \simeq \frac{n_1\Delta}{c} \frac{(u-2)}{(u+2)} \frac{2}{(2.5)^{4/(u+2)}} \frac{(m+2.5)^{4/(u+2)}_{\max}}{(m+1)^2_{\max}}. \quad (36)$$

Equation (29) can be used to eliminate the last factor in (36). For the first term of (34), representing the major response due to $a_u x^u$ in (1), the maximum value of t is

$$t_{\max(u)} = \frac{n_1\Delta}{c} \frac{(u-2)}{(u+2)}. \quad (37)$$

For the second term of (34), representing the perturbing term $a_r x^r$ in (1), the maximum value of t is

$$t_{\max(r)} = \frac{n_1 \Delta}{c} \frac{\delta}{(1 - \delta)^{r/(u+2)}}. \tag{38}$$

Equation (34) is not valid for $u = 2$; for the later case the impulse response is

$$P(t)|_{u=2} = \frac{\pi}{r} t^{(4-r)/r} \left\{ \frac{4096(1 - \delta)^{r/4} m_{\max}^{r/2}}{32^{r/2}(41.254)^{(2-r)/2}(2r - 4)\delta\Delta} \right\}^{(2r-4)/r}. \tag{39}$$

IV. THE NEAR-PARABOLIC INDEX DISTRIBUTION

Letting $u = 2$ in the equations of the preceding section yields the near-parabolic index distribution. As shown by (35), the impulse response has zero width when $a_r = 0$ in the approximation made here. In the cylindrical fiber there is no index distribution which gives zero delay distortion among all the various modes. There is a distribution of index which minimizes the delay distortion, and we now evaluate this condition. We can use the above theory to evaluate the cylindrical waveguide by noting the two limiting conditions already known for low dispersion. Take the form

$$n = n_1(1 - \tfrac{1}{2}\alpha^2 R^2 + b_4 \alpha^4 R^4 + \cdots) \tag{40}$$

in which R is the radial transverse coordinate. In two earlier papers[6,13] it has been pointed out that the value of b_4 must be different to produce no dispersion for meridional rays versus skew rays; the difference in b_4 was found to be $\frac{1}{6}$. Kawakami and Nishizawa[6] found that b_4 must be $\frac{1}{2}$ to give no dispersion for skew rays and must be $\frac{1}{3}$ to give no dispersion for meridional rays. We can visualize minimizing the dispersion for one ray type, and thus experiencing a maximum dispersion corresponding to a change in b_4 of $\frac{1}{6}$. We do this in the approximation used here by setting $u = 2$, $r = 4$, and making

$$a_4 = -\tfrac{1}{3}a_2^2 \tag{41}$$

corresponding to no dispersion for meridional rays. From (24), (25), (31), and (41) we find for this "ideal" index distribution in the round fiber the specific group delay

$$\tau = \frac{n_1}{c}\left\{1 - 0.26 \frac{(m + 1)^2}{(m + 1)_{\max}^2} \Delta^2\right\} \tag{42}$$

and

$$t_{\max} = -0.26 \frac{n_1}{c} \Delta^2. \tag{43}$$

This agrees reasonably well with tbe value $-n_1\Delta^2/8c$ arrived at by Gloge and Marcatili[10] using an entirely different analysis for an optimum index distribution defined differently.* The rms widtb of the impulse response corresponding to (41) and (42) is

$$\sigma = 0.752 \frac{n_1}{c} \Delta^2. \tag{44}$$

In contrast with this, the simple step-index fiber [represented by $a_r = 0$ and $n \to \infty$ in (1)] has an rms impulse response width of

$$\sigma = \frac{1}{\sqrt{12}} \frac{n_1}{c} \Delta. \tag{45}$$

Thus the "ideal," corresponding to (42) and (43), is smaller by a factor of 0.26Δ. Since $\Delta \simeq 0.01$ the ideal graded-index fiber has an impulse response narrower by a factor of about 400 than that for the conventional fiber.

The fourth-order term represented by (41) corresponds to

$$\delta = -\tfrac{2}{3}\Delta. \tag{46}$$

This is very little different from the simple parabola—not enougb to see on Fig. 1 where curve 1 represents *both* $\delta = 0$ and (46) for $\Delta \simeq 0.01$. More importantly, (46) and (41) imply that the fourth-order term decreases in size relative to tbe second-order term as Δ decreases. The inaccuracies of material processing are likely to prevent this as Δ becomes small. A more probable limit is a fixed value of δ in (25) as Δ changes. Tbis results in a specific group delay

$$\tau = \frac{n_1}{c} \left\{ 1 + \frac{0.389\delta}{(1-\delta)} \frac{(m+1)^2}{(m+1)^2_{\max}} \Delta \right\} \tag{47}$$

and an rms width for the impulse response

$$\sigma = 0.112 \frac{n_1}{c} \Delta \frac{\delta}{(1-\delta)}. \tag{48}$$

* If instead of minimizing the delay distortion for either the skew rays or meridional rays we had minimized the delay distortion for an index equation (40) at the *mean* of the index values giving minimum distortion for the skew and meridional rays—i.e., at $b = 5/12$—then the maximum departure in b for any mode corresponds to a change in $b = \pm 1/12$. This corresponds to $a_4 = \pm 1/6a_2^2$, leading to

$$\tau = \frac{n_1}{c} \left\{ 1 \pm 0.13 \frac{(m+1)^2}{(m+1)^2_{\max}} \Delta^2 \right\}$$

in place of (42). The coefficient 0.13 corresponds almost exactly to the result of Gloge and Marcatili,[10] but the present analysis indicates twice the total delay spread predicted by Gloge and Marcatili due to the $\pm$ sign.

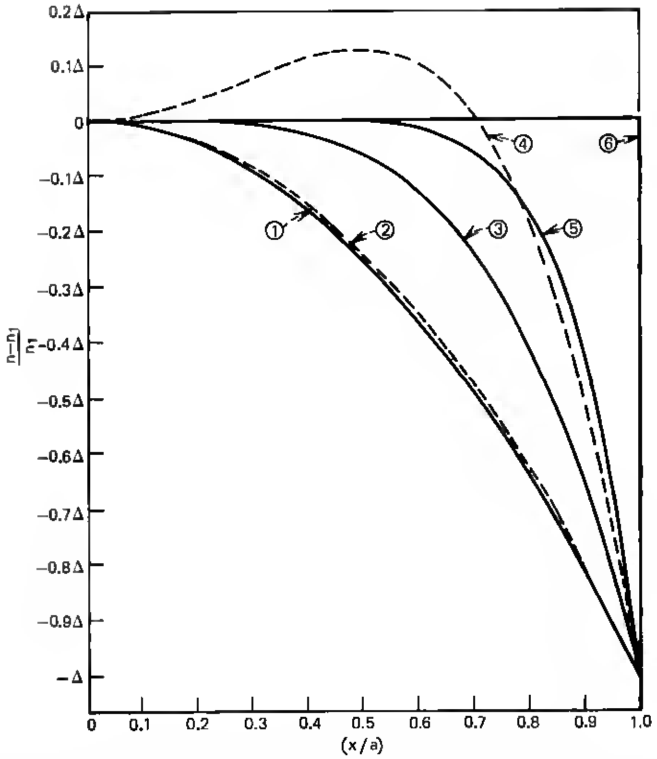

Fig. 1—Normalized index of refraction versus transverse coordinate (x/a) for the following parameters in eqs. (1) and (25): curve ①, $u = 2$, $\delta = 0$; curve ②, $u = 2$, $r = 4$, $\delta = 0.05$; curve ③, $u = 4$, $\delta = 0$; curve ④, $u = 4$, $r = 2$, $\delta = -1$; curve ⑤, $u = 8$, $\delta = 0$; curve ⑥, $u = \infty$, $\delta = 0$.

For $\delta = 0.05$, σ becomes $0.00591\ n_1\Delta/c$ which is narrower than for the step-index fiber by a factor of about 50 independent of Δ.

We note from (39) that the impulse response for the ideal index distribution perturbed by a fourth-order term ($r = 4$) is a rectangular pulse, shown as curve ② in Fig. 2. However, if the perturbation were sixth order, $r = 6$, the impulse response would vary as $t^{-\frac{1}{2}}$. Other values of r give other impulse-response shapes, which we discuss further in the next section.

Finally we note from (47) that the impulse response due to the fourth-order perturbation of the ideal distribution may either lead or lag the impulse at $\tau = n_1/c$, depending on whether δ is positive or negative [see (1) and (25)]. Similar effects due to the sign of δ are

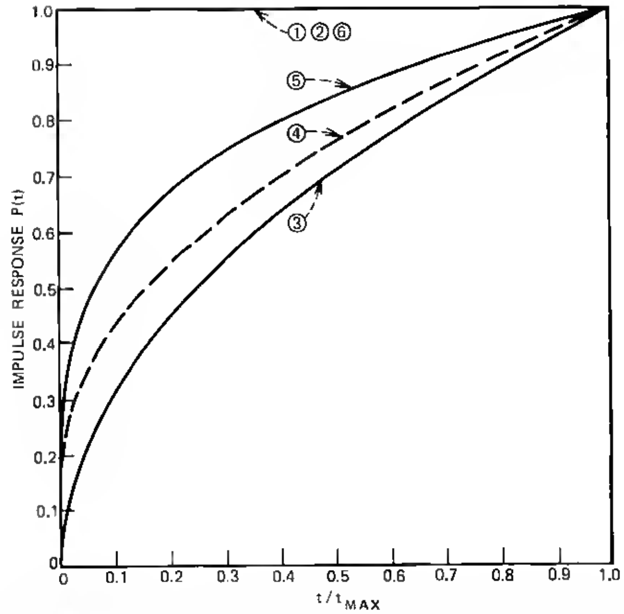

Fig. 2—Impulse response $P(t)$ versus $t/t_{\max}$ where $t = \tau - n_1/c$ and τ is given by (31) and (33). The conditions are the same as defined under Fig. 1.

found for perturbations of the nonideal index distributions and may be seen in (31).

In Section VI there is a discussion of Fig. 8 whicb shows the effects of perturbing the parabolic index distribution in several ways.

V. DISCUSSION OF THE INDEX DISTRIBUTION, $n = n_1(1\text{-}\frac{1}{2}a_u x^u)$

In this section we discuss the distributions obtained by setting $a_r = 0$ in (1) whicb mean $\delta = 0$ in (24) and (25).

Tbe total spread in specific group delay for all modes $t_{\max,u}$ is given by (37), which gives a null value when $u = 2$. As already discussed, this is a simplification in the vicinity of the "ideal" distribution. However, (37) gives a valid representation as u departs significantly from the value 2. Figure 3 shows the variation in $t_{\max,u}$ versus u. The bebavior in the vicinity of $u = 2$ is a form of singularity. For 5-percent error in u from tbe ideal, $t_{\max,u} \simeq 0.025 n_1 \Delta / c$. Tbis compares with $0.0244 n_1 \Delta / c$ for tbe modal delay spread from (47) for 5-percent (at $x = a$) fourth-order perturbation of the ideal. We conclude that it is not particularly important how the ideal is perturbed.

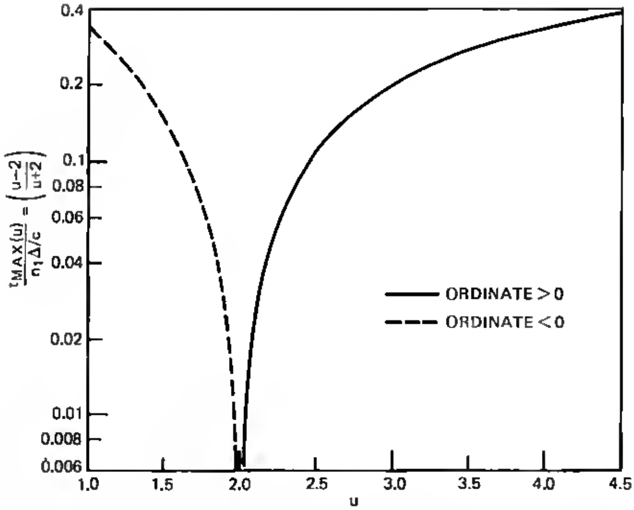

Fig. 3—Normalized modal group-delay spread versus u for $a_r = 0$ in eq. (1).

Tbe value of $t_{\max,u}$ from (37) and plotted in Fig. 3 is not very sensitive to tbe value of u at values removed from $u = 2$. The reduction in rms width of the impulse response is illustrated in Fig. 4. For $u = \infty$ (the conventional step-index fiber) the value of $\sigma/(n_1\Delta/c)$ is $1/\sqrt{12}$ or 0.289. This is reduced by a factor of 2 for u near 6, and by a factor of 4 for u near 3.5. These results are identical to those of Gloge and Marcatili.[10]

The sbape of tbe impulse response, given by (39) for $u = 2$ and by (34) for u away from 2, is plotted in Fig. 2 for several cases of interest.

The number of modes wbich can propagate, given by (28) for tbe square fiber, is plotted as a function of u^{-1} in Fig. 5. For comparison the corresponding quantity for the round fiber from Ref. 10 is also plotted. Tbe ratio is $4/\pi$ at $u = \infty$, and near 2 for $u = 2$. The approximations made here are seen to be good, though not perfect.

VI. PERTURBATIONS OF THE GENERAL INDEX DISTRIBUTION

We discuss now some of the results for tbe perturbed index distributions, eq. (1). We recall the solutions bave been obtained assuming $|a_r| \ll a_u$ or $|\delta| \ll 1$.

The solutions for tbe specific group delay, given in (31) and (33), contain the quantity Q as a factor in tbe perturbing term. Tbe factor

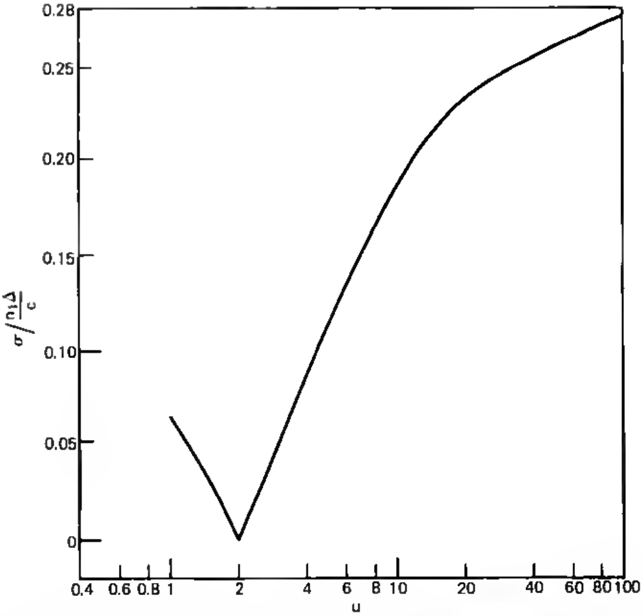

Fig. 4—Normalized rms width of the impulse response versus u for $a_r = 0$ in eq. (1).

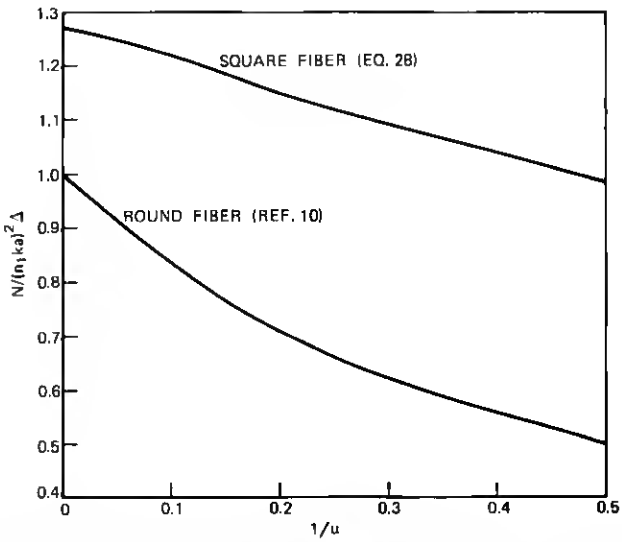

Fig. 5—Total number of propagating modes versus u^{-1} for $a_r = 0$ in eq. (1).

Q, given by (32), contains the principal effect of the exponents u and r on specific group delay. Figure 6 shows how Q varies with r for the special case $u = 2$. The region very near $r = 2$ is in question since we know the "ideal" index distribution differs slightly from $u = 2$. Elsewhere, the results should be significant and may be used in (34), (31), and (33).

More general curves for Q are given in Fig. 7. We observe that when $r > 0$ the maximum value of Q is not very dependent on u but the most sensitive region of r (giving largest Q) does depend somewhat on u. An intuitive feel for the changes in the index distribution which correspond to some of the curves in Fig. 7 can be obtained by examination of Fig. 8. Figure 8 shows the normalized index n versus transverse coordinate (x/a). The curve labeled $r = 2$ corresponds to the pure parabolic distribution. The other curves correspond to $\delta = 0.05$ with various values of r in the perturbing term and u always equal to 2.

We note in Fig. 7 that, at $u = 2$, the value of Q at $r = 10$ is much larger than at $r = \infty$. This may seem surprising, since a step-index change occurs at $(x/a) = 1.0$ when $r = \infty$. Below $(x/a) = 1$ the $r = \infty$ curve in Fig. 8 corresponds to a pure parabolic gradient between the ordinate equal zero and -0.95Δ.

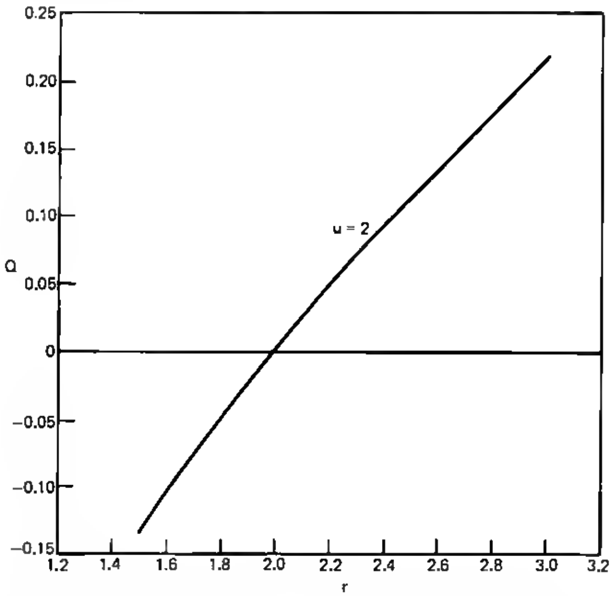

Fig. 6—Q versus r for the near-parabolic index distribution.

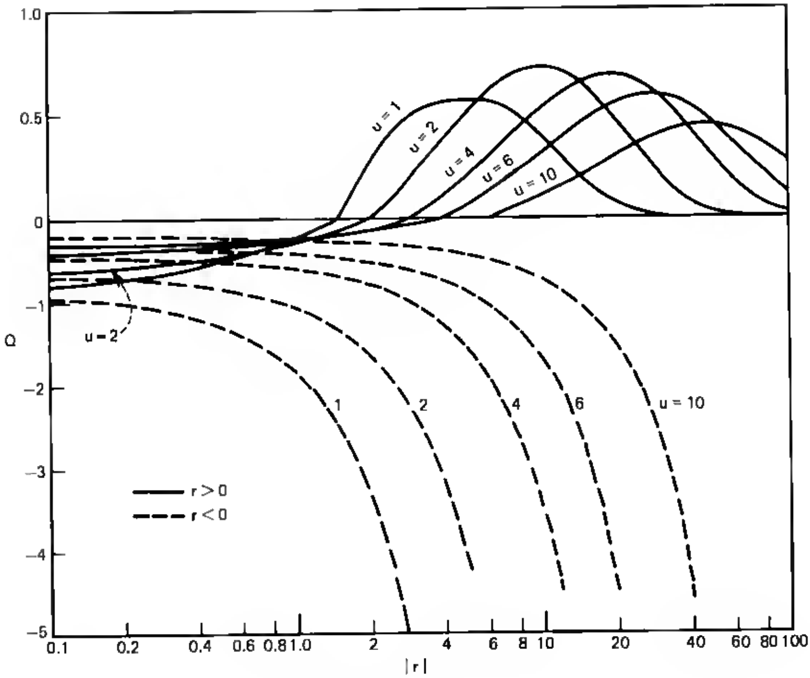

Fig. 7—Q versus $|r|$ with u as a parameter.

We also note in Figs. 8 and 7 that dips in the index distribution near (x/a) equal zero (curves for $r = -0.4$ and -0.1) yield values of $|Q|$ comparable to those for r in the range 4 to 20.

VII. CONCLUSION

The above analysis provides an approximate solution for the delay distortion to be expected in a wide variety of graded-index fibers, representable by (1) witb $|a_r| \ll a_u$.

In general, gradual tapering of the index between the center of the fiber and the outer support provides reduced delay distortion. Only in the vicinity of the near-parabolic distribution is the performance highly sensitive to the exact index distribution. The "ideal" near-parabolic distribution provides a potential reduction in delay distortion of several hundred times compared to the step-index distribution of the conventional clad fiber. With an accuracy of the order of 5 percent in achieving the "ideal" distribution, the reduction in delay distortion is on the order of 50.

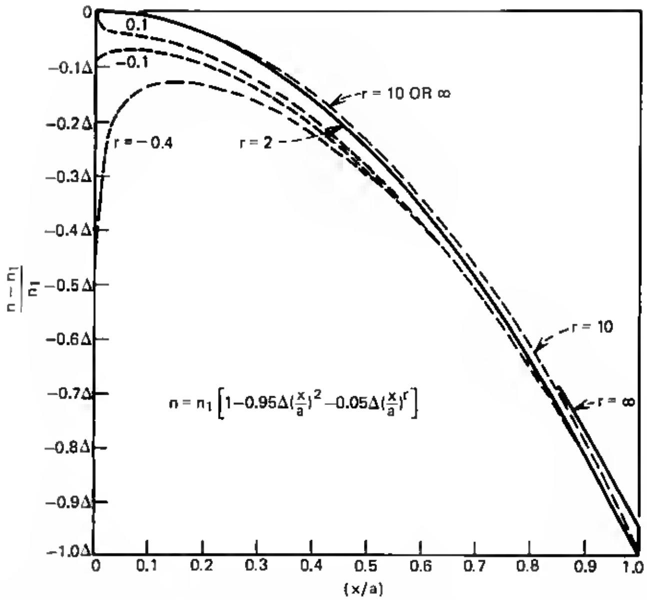

Fig. 8—Normalized index versus transverse coordinate (x/a) for several perturbations of the parabolic index distribution.

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